Method to Optimize Nacelle Shape in a Supersonic Cruise Aircraft

Reiner Suikat* and Saeed Farokhi†
University of Kansas, Lawrence, Kansas

A computer program is developed to optimize the shape of a nacelle installed in a supersonic aircraft for minimum drag. The program is also capable of optimizing the wing camber of the same aircraft. As a unique feature, the present code accounts for the aerodynamic forces on the entire airplane in contrast to previous wing camber optimization codes that included only the wing forces. The program is based on a panel-method analysis code by Woodward, and the accuracy of the program is checked with the available wind-tunnel data on isolated components as well as full configurations. The computed results are in general agreement with the available data. The results of several optimization test runs are presented and show agreement with trends predicted by other researchers based on theoretical and experimental studies. At higher angles of attack, when supersonic vortex lift becomes significant, the analysis code underpredicts the aircraft lift coefficient.

Nomenclature

A = influence coefficient matrix
a_{ij} = influence coefficient
c = chord length
c_D = drag coefficient
c_L = lift coefficient
c_M = pitching moment coefficient
c_p = pressure coefficient
D = drag
F = force
L = lift
M = pitching moment, Mach number
N = number of panels
n = induced velocity normal to panel surface
q = dynamic pressure
S = panel surface area
s = singularity strength
t = wing thickness
U, V, W = influence coefficient matrices in x, y, and z directions
u, v, w = velocity components in the x, y, and z directions
x, y, z = coordinate axes
α = angle of attack
γ = ratio of specific heats
Δ = displacement
δ = panel incidence angle
λ = Lagrangian multiplier
ϕ = velocity potential
θ = panel orientation, rotation about x axis

Subscripts
B = body
des = design
i, j = panel numbers, summation indices
k, l = summation indices
N = nacelle
S = sources
W = wing

Superscripts
$+$ = change in quantity due to optimization

Received Aug. 31, 1987; presented as Paper 87-2865 at the AIAA/AHS/ASEE Aircraft Design Systems and Operations Meeting, St. Louis, MO, Sept. 14–16, 1987; revision received Dec. 15, 1987.

*Graduate Research Assistant. Student Member AIAA.
†Associate Professor, Aerospace Engineering. Member AIAA.

Introduction

ONE of the primary goals in designing a supersonic cruise aircraft is to minimize the airplane drag. In this paper, a method is presented that optimizes the external shape of a nacelle in order to reduce the pressure drag on the airplane. The method is based on a low-order panel-method analysis code, which provides efficient computation and is user friendly. Panel methods are a well-established computational tool in airplane aerodynamic analysis and design. However, their use in design has so far been limited to wing camber design. The extension of these design capabilities to other components of an aircraft, including the entire configuration, should provide the designer with a very useful analytical tool. The present computer program is called Subsonic and Supersonic Configuration Analysis and Design (SSCAD).

The proposed method of analysis accounts only for the external nacelle flow; hence, it excludes the internal flow as well as the exhaust jet/airframe interaction. Therefore, the method is restricted to supersonic flows requiring the Mach cone originating from the exhaust plane not to intersect the airplane. The computation is based on the linearized potential flow theory, which excludes viscous effects. These assumptions provide faster computation, but they also limit the accuracy of the results. Hence, the leading-edge suction force and the vortex lift phenomenon, which occur in subsonic flow and in supersonic flow with subsonic leading edge, cannot be captured with this method.

A detailed description the theory and more computational validation of this research is found in Ref. 1.

Computer Code

The computer program described in this paper is based on Woodward's low order panel-method code[2] and incorporates several improvements from the Unified Subsonic and Supersonic Aerodynamics (USSAERO) program, a later low-order panel-method program by Woodward.[3] This code has been chosen because the low-order formulation allows fast computation and also leads to a set of linear equations for the optimum singularity distribution, which can then be easily transformed into the optimum shape.

The present code is capable of computing the aerodynamic forces acting on complete wing-body-nacelle configurations.

The wing may consist of up to six trapezoidal sections with arbitrarily specified thickness, camber, and twist distributions. The singularity panels are located in the mean wing plane. The wing sections need not be connected; therefore, multisurface configurations including vertical and horizontal tail can be modeled. The body is restricted to circular cross-section. However, the body radius may vary arbitrarily so that area-ruled bodies can be analyzed. The nacelle can be of any arbitrary shape and located anywhere on the configuration. Inlet bleed may also be modeled by source singularities of specified strength.

The underlying principle of a panel-method program is as follows. First, the configuration is divided into a large number of surface panels. Singularities of certain types are then distributed over those panels. In a low-order method these singularities are assumed to be of constant strength over each panel. In other methods, like PANAIR, a second-order method, the singularities are allowed to vary across each panel, in this case quadratically. The types of singularities used in the present method are surface sources on all panels to account for the airplane volume and surface vortices distributed over the wing only to account for the wing lift. The influence, i.e., the induced flow, of these singularities on other panels is described by "influence coefficients." An influence coefficient a_{ij} is defined as the velocity component in a specified direction on panel i induced by a unit strength singularity on panel j. Since the computation is based on the inviscid-potential flow theory, the influence coefficients are derived from solutions of the Prandtl-Glauert equation:

$$(1-M^2)\,\phi_{xx}+\phi_{yy}+\phi_{zz}=0 \tag{1}$$

The singularity strengths are then established by satisfying the surface boundary conditions. The present method utilizes the von Neumann boundary condition, which provides for a tangential flow on the surface, i.e., the normal flow component is equal to zero on a solid surface. This condition must be satisfied at one point on each panel, called the control point. Following this technique, one obtains N equations for the N unknowns, where N is the number of panels. The boundary equations can be written in matrix form:

$$As=[\delta_i-\alpha\cos\theta_i]-a_{\text{thick}} \tag{2}$$

where a_{thick} is the normal velocity induced by the wing source.

Since the wing sources are computed directly from the wing thickness distribution,[4] the right-hand side of the equation contains only known quantities and the system of equations can be solved for the unknown singularity strengths. The next step is to compute the total induced velocities in the x, y, and z directions by summing the velocities induced by each singularity. For example, the velocity components in the x direction are obtained by

$$u=Us+u_{\text{thick}} \tag{3}$$

where u_{thick} is the x component of the velocities induced by the wing thickness. Then the panels' pressure coefficients can be computed using the thin-wing approximation

$$c_p=-2u \tag{4a}$$

or the exact isentropic equation

$$c_p=\frac{-2}{\gamma M^2}\left\{\left[1+\frac{\gamma-1}{2}M^2\,(l-q^2)\right]^{3.5}-1\right\} \tag{4b}$$

It is observed, however, that the exact isentropic equation does not always produce more accurate results than the thin-wing approximation. This is especially the case for highly swept wings in supersonic flow. After the pressure coefficients are calculated, the aerodynamic forces acting on the airplane are computed by summing the forces acting on each panel, where the pressure coefficients are assumed to be constant on any given panel. The force acting normal to the panel surface is given by

$$F=c_p\,S \tag{5}$$

The normal and axial forces are the components of this force in the body axes x and z direction. In order to obtain the force coefficients, these forces are rotated into the stability axes and nondimensionalized.

Design Procedure

The nacelle shape is optimized in the following way. First, an optimum singularity distribution is found. The optimum distribution is the one that generates the lowest airplane pressure drag. Then panel incidence angles corresponding to this distribution are computed and integrated over the length of the nacelle to obtain the nacelle shape.

Several important assumptions are made to obtain an efficient algorithm. First, is is assumed that the changes in panel slopes and locations are small, so that the influence coefficients remain unaffected. Second, the thin-wing approximation for the pressure coefficient [Eqs. (4)] will be used on all panels. Then, the final system of equations for the optimal singularity distribution is linear and, therefore, easy to solve.

Certain constraints are applied to the design procedure. The design point is specified by the flight Mach number M angle of attack α and the design lift coefficient c_L. A design pitching moment coefficient c_M can also be specified to avoid generating additional trim drag. The second constraint is geometric in nature. For the nacelle design, the inlet and exhaust cross-sections are required to stay fixed. To avoid excessive shrinkage of the nacelle, one additional station along the nacelle axis can be required to stay fixed. The nacelle shape optimization is performed by computing changes in panel slopes and then integrating those over the length of the nacelle to obtain the shape. Therefore, the geometric constraints can be satisfied by requiring the integral of the changes in slope over the length of the nacelle to be equal to zero.

$$\Delta=\sum_{i=1}^{N_{\text{strip}}}\delta_i^+\,c_i=0 \tag{6}$$

where N_{strip} is the number of panels along the nacelle, c_i the length of panel i, and δ_i^+ the change in slope of panel i. The problem of finding the optimal singularity distribution is solved via the method of Lagrangian multipliers. Consider the function

$$F=D+\lambda_1(L-L_{\text{des}})+\lambda_2\,(M-M_{\text{des}})+\sum_{j=1}^{N_{\text{rows}}}\lambda_{j+2}\sum_{i=1}^{N_{\text{strips}}}c_i\delta_i^+ \tag{7}$$

where N_{rows} is the number of panel rows around the nacelle. This function has a minimum if the total drag D is minimized and the constraint equations are exactly satisfied. To find the minimum, it is necessary to express all terms in the equation in terms of the unknown singularity distribution on the nacelle. The minimum can then be obtained by setting the derivatives of F with respect to the singularity strengths and the Lagrangian multipliers equal to zero, i.e.,

$$\frac{\partial F}{\partial s_{Ni}}=\frac{\partial D}{\partial s_{Ni}}+\lambda_1\frac{\partial L}{\partial s_{Ni}}+\lambda_2\frac{\partial M}{\partial s_{Ni}}+\sum_{j=1}^{N_{\text{rows}}}\lambda_{j+2}\sum_{k=1}^{N_{\text{strips}}}c_k\frac{\partial\delta}{\partial s_{Nk}}=0 \tag{8}$$

$$
\left[\begin{array}{cccc|c|cccc}
\frac{\partial^2 D}{\partial s_{N1}\partial s_{N2}} & \frac{\partial^2 D}{\partial s_{N1}\partial s_{N2}} & \cdots & \frac{\partial^2 D}{\partial s_{N1}\partial s_{NN}} & \frac{\partial L}{\partial s_{N1}} & \frac{\partial \Delta_1}{\partial s_{N1}} & \frac{\partial \Delta_2}{\partial s_{N1}} & \cdots & \frac{\partial \Delta_m}{\partial s_{N1}} \\
\frac{\partial^2 D}{\partial s_{N2}\partial s_{N1}} & & & \vdots & \frac{\partial L}{\partial s_{N2}} & \frac{\partial \Delta_1}{\partial s_{N2}} & & & \vdots \\
\vdots & & & \vdots & \vdots & \vdots & & & \vdots \\
\frac{\partial^2 D}{\partial s_{NN}\partial s_{N1}} & \cdots & \cdots & \frac{\partial^2 D}{\partial s_{NN}\partial s_{NN}} & \frac{\partial L}{\partial s_{NN}} & \frac{\partial \Delta_1}{\partial s_{NN}} & \cdots & \cdots & \frac{\partial \Delta_m}{\partial s_{NN}} \\
\hline
\frac{\partial L}{\partial s_{N1}} & \frac{\partial L}{\partial s_{N2}} & \cdots & \frac{\partial L}{\partial s_{NN}} & 0 & \cdots & \cdots & \cdots & 0 \\
\hline
\frac{\partial \Delta_1}{\partial s_{N1}} & \frac{\partial \Delta_1}{\partial s_{N2}} & \cdots & \frac{\partial \Delta_1}{\partial s_{NN}} & \vdots & & & & \vdots \\
\frac{\partial \Delta_2}{\partial s_{N1}} & & & \vdots & \vdots & & & & \vdots \\
\vdots & & & \vdots & \vdots & & & & \vdots \\
\frac{\partial \Delta_m}{\partial s_{N1}} & \cdots & \cdots & \frac{\partial \Delta_m}{\partial s_{NN}} & 0 & \cdots & \cdots & \cdots & 0
\end{array}\right]
\left[\begin{array}{c}
s_{N1} \\ s_{N2} \\ \vdots \\ s_{NN} \\ \hline \lambda_0 \\ \lambda_1 \\ \vdots \\ \vdots \\ \lambda_m
\end{array}\right]
=
\left[\begin{array}{c}
-\frac{\partial D}{\partial s_{N1}}\Big|_{const} \\ -\frac{\partial D}{\partial s_{N2}}\Big|_{const} \\ \vdots \\ -\frac{\partial D}{\partial s_{NN}}\Big|_{const} \\ \hline L_{des} - L_{const} \\ -\Delta_{1const} \\ \vdots \\ \vdots \\ -\Delta_{mconst}
\end{array}\right]
$$

Fig. 1 Optimization matrix equation.

where $i = 1,2,\ldots, N_N$ and

$$\frac{\partial F}{\partial \lambda_1} = L - L_{\text{des}} = 0$$

$$\frac{\partial F}{\partial \lambda_2} = M - M_{\text{des}} = 0$$

$$\frac{\partial F}{\partial \lambda_j} = \sum_{i=1}^{N_{\text{strip}}} c_i\,\delta_i^+ = 0, \; j = 3,4,\ldots,N_{\text{rows}}+2$$

This is a set of $N_N + N_{\text{rows}} + 2$ equations that can be solved for the unknowns s_N and λ_j. The equations for the lift and drag forces as well as their derivatives are derived from the equation for the force [Eq. (5)] with c_p given by the thin wing approximation [Eqs. (4)]. The velocity component u is expressed in terms of the singularity distributions with the help of Eq. (3). Then the boundary equations are split into submatrices to relate the singularity strengths on the body and the wing to those on the nacelle

$$
\begin{bmatrix} A_{NN} & A_{NW} & A_{NB} \\ A_{WN} & A_{WW} & A_{WB} \\ A_{BN} & A_{BW} & A_{BB} \end{bmatrix}
\begin{bmatrix} s_N \\ s_W \\ s_B \end{bmatrix}
\begin{matrix} = \\ = \\ = \end{matrix}
\begin{bmatrix} -n_N + \left[\delta_i^+\left(1+u_{NS_i}\right)\right] \\ -n_W \\ -n_B \end{bmatrix}
\tag{9}
$$

where

n_X = normal velocity due to panel slope, angle of attack, wing sources, and bleed, and

u_{NS_i} = u component of velocity induced on nacelle panels by sources and bleed.

The equations are then put into matrix form as shown in Fig. 1. In this case, the row and column corresponding to the optional moment constraint has been omitted. Note that the optimization matrix is symmetric. This system of equations is solved using a standard routine for solving a system of linear equations. The solutions are the Lagrangian multipliers and the singularity distribution that will yield the minimum drag. The changes in slope applied to the nacelle panel are obtained by solving Eq. (9) for δ_i^+.

$$[\delta_i^+\ (1+u_N)] = AR\ s_N + r_N \tag{10}$$

where

$$AR = A_{NN} - DN2\ CN^{-1}\ DN1$$

$$r_N = -DN2\ CN^{-1}\ n_{WB} + n_N$$

$$DN1 = \begin{bmatrix} A_{WN} \\ A_{BN} \end{bmatrix}; \; DN2 = [A_{NW} \quad A_{NB}]$$

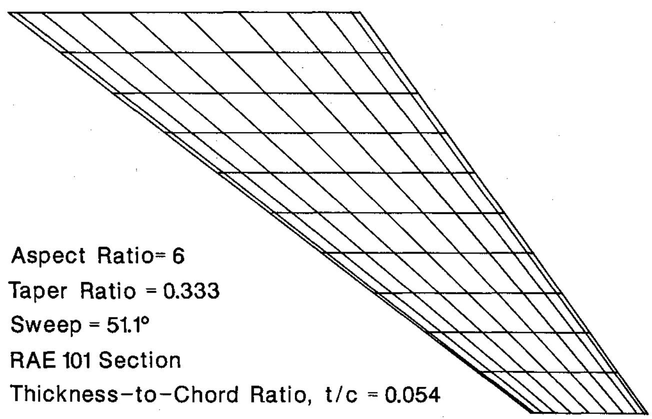

Fig. 2 Royal Aircraft Establishment (RAE) wing C.

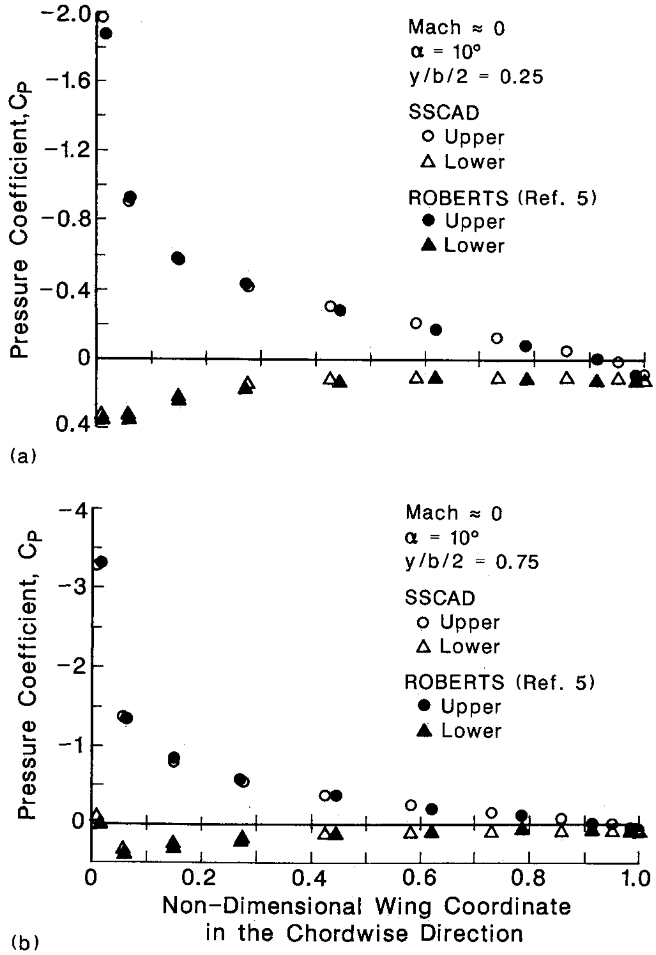

Fig. 3 Comparison of theoretical pressure distributions on RAE wing obtained with SSCAD and Roberts and Rundle method.[5]

$$CN = \begin{bmatrix} A_{WW} & A_{WB} \\ A_{BW} & A_{BB} \end{bmatrix};\ n_{WB} = \begin{bmatrix} n_B \\ n_W \end{bmatrix}$$

Integrating the new slopes over the length of nacelle then gives the desired optimized shape of the nacelle.

Computational Results

Two sample analysis cases are presented here to demonstrate the accuracy of the code. First, the pressure distribution on a tapered wing with 51 deg leading-edge sweep angle computed at low Mach number, i.e., incompressible flow, and $\alpha = 10$ deg is compared to results obtained with the Roberts and Rundle method,[5] which is regarded as one of the most accurate potential flow solvers.[6] It can be seen that the SSCAD results agree extremely well with the Roberts and Rundle method results. The geometry of this wing is shown in Fig. 2 and the computed pressure distributions in Fig. 3. The second example is a NASA supersonic cruise arrow-wing configuration,[7] for which the longitudinal aerodynamic char-

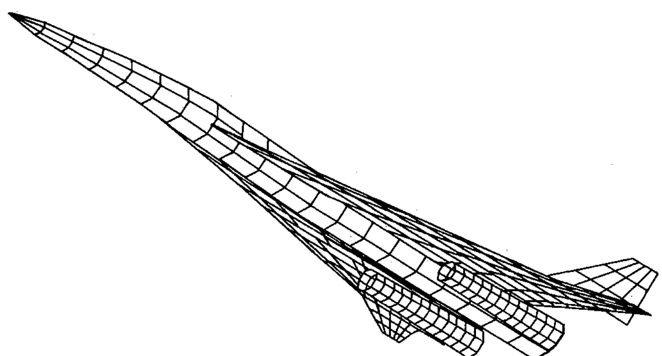

Fig. 4 Panel layout on the supersonic cruise airplane.

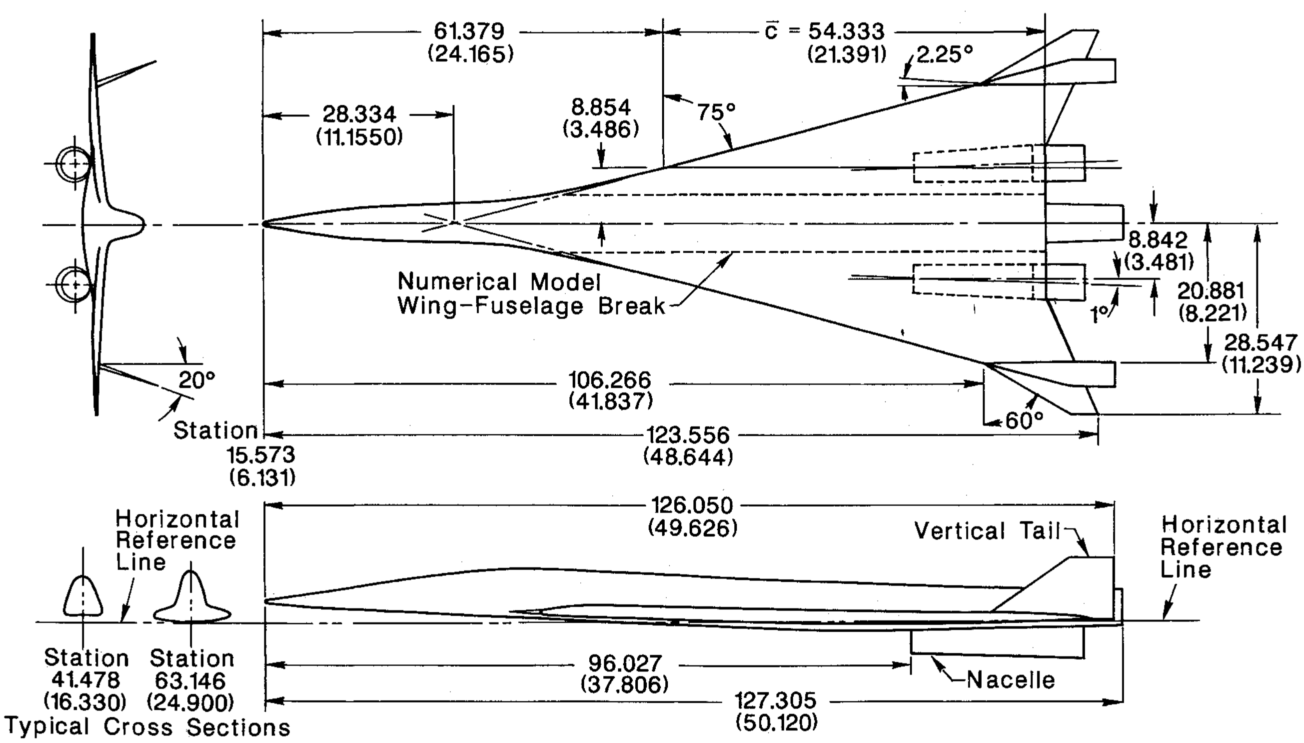

Fig. 5 Three-view sketch of a supersonic cruise model[7]; all linear dimensions in cm (in.).

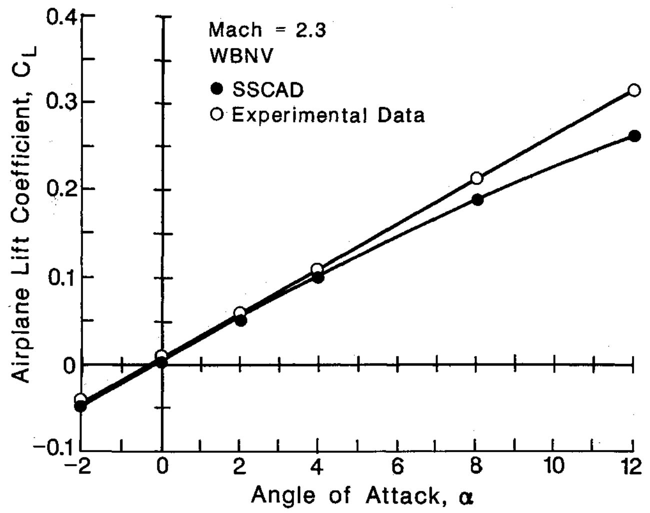

Fig. 6 Lift curve of supersonic cruise model.

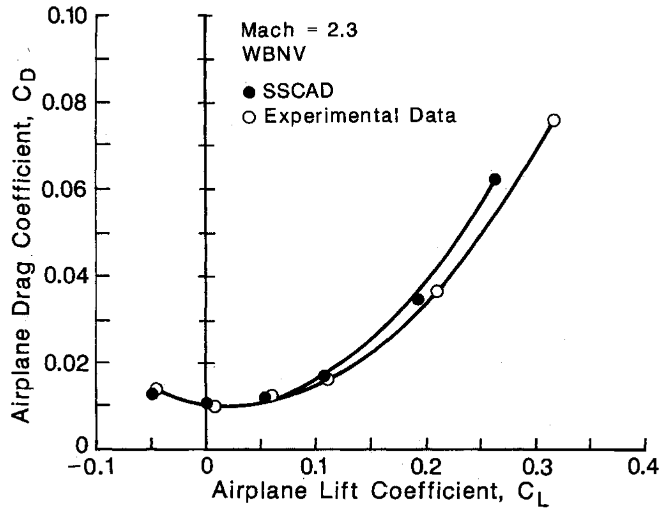

Fig. 7 Drag polar comparison between SSCAD and experimental data.

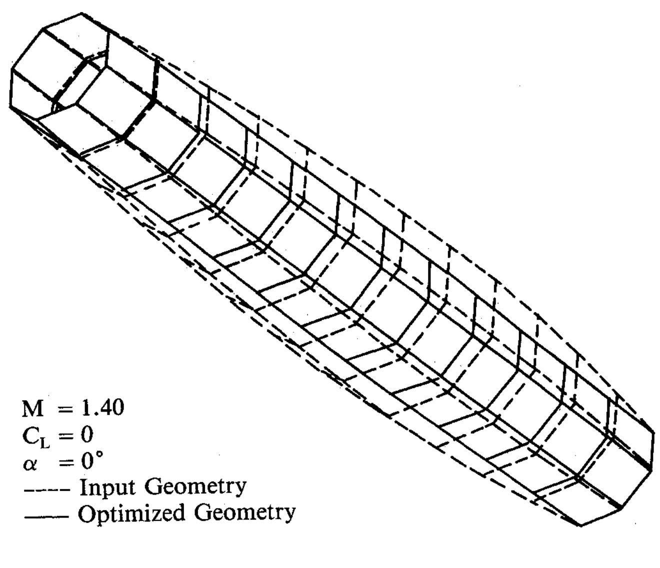

Fig. 8 Unconstrained isolated nacelle design (with inlet and exhaust areas fixed).

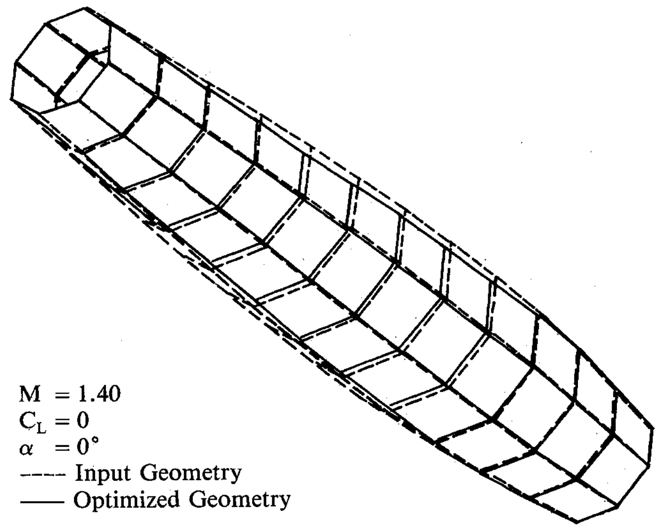

Fig. 9 Constrained isolated nacelle design (with inlet, exit, and maximum cross-sectional areas fixed).

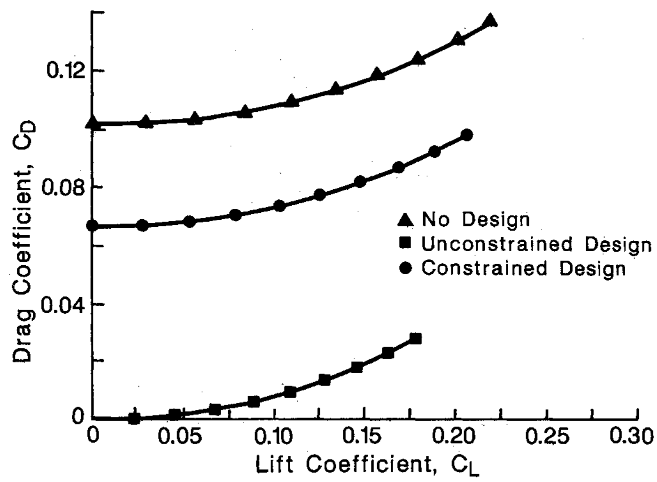

Fig. 10 Drag polars for isolated nacelles.

acteristics are computed. Figure 4 depicts the panel layout for this airplane and a scaled drawing is given in Fig. 5. The lift curve and drag polar, shown in Figs. 6 and 7, display an underprediction of the lift coefficient by SSCAD at high angle of attack. This may be caused by the vortex flow or numerical problems of the basic panel method code. As Maskew finds,[8] panel methods using the von Neumann boundary condition tend to become unstable when two surfaces are opposing each other at close distance. This is the case for the nacelle/wing and also the vertical tail/wing interference. However, the computed results are still very accurate for small angles of attack.

To demonstrate the versatility of the present method, several design cases are presented here. First, an isolated nacelle is optimized at $M = 1.4$ and zero angle of attack. This example demonstrates the use of the optional intermediate cross-section constraint. In the first run, only the basic constraint of inlet and exhaust shape is used. Therefore, SSCAD returns the cylindrical shape as the optimum solution (Fig. 8), which indeed has the lowest pressure drag, namely zero. However, this design may be useless, because the nacelle has to house an engine, which requires a certain space. Hence, in the second run, the maximum diameter of the original nacelle is constrained and the changes in shape are now very small (Fig. 9). The drag polars for the original nacelle and for

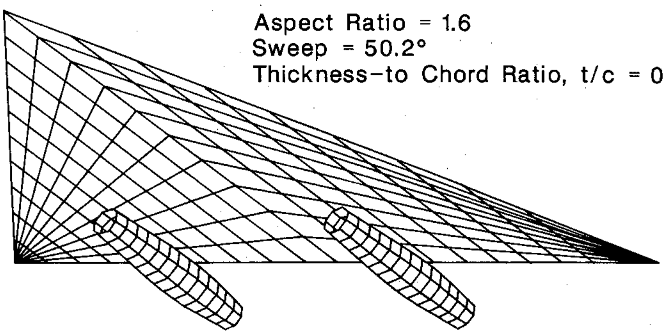

Fig. 11 Delta wing-nacelle configuration.

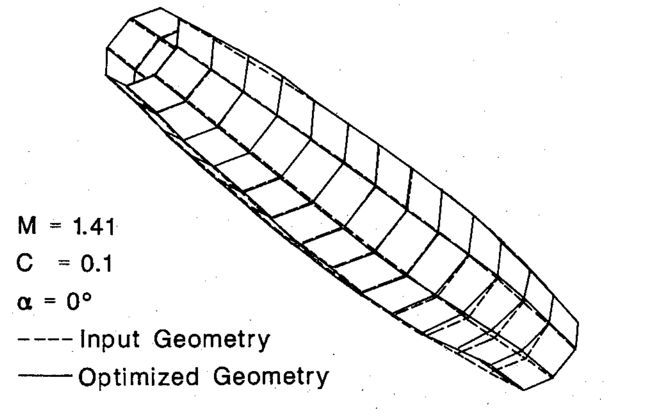

Fig. 14 Optimized nacelle geometry with inlet, exit, and maximum cross-sectional areas fixed.

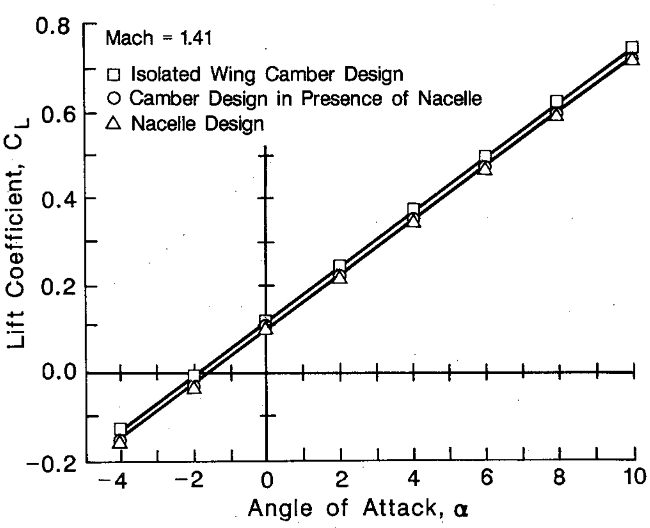

Fig. 12 Lift curves of wing-nacelle configuration.

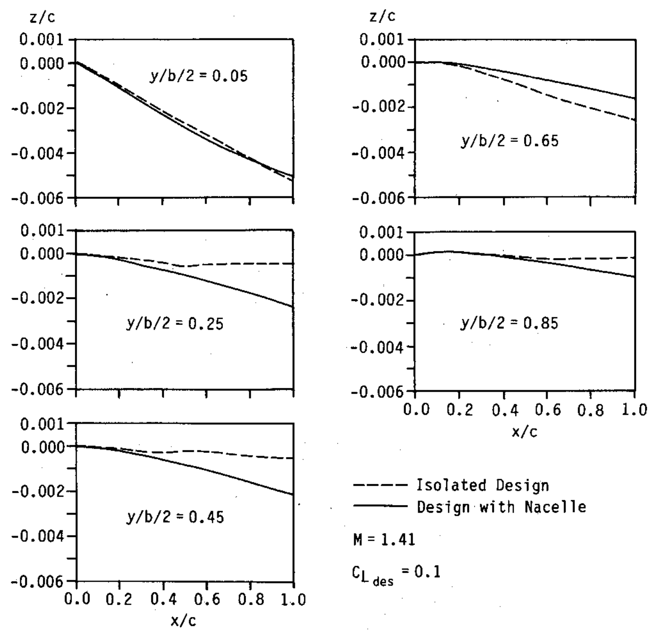

Fig. 15 Optimized wing camber.

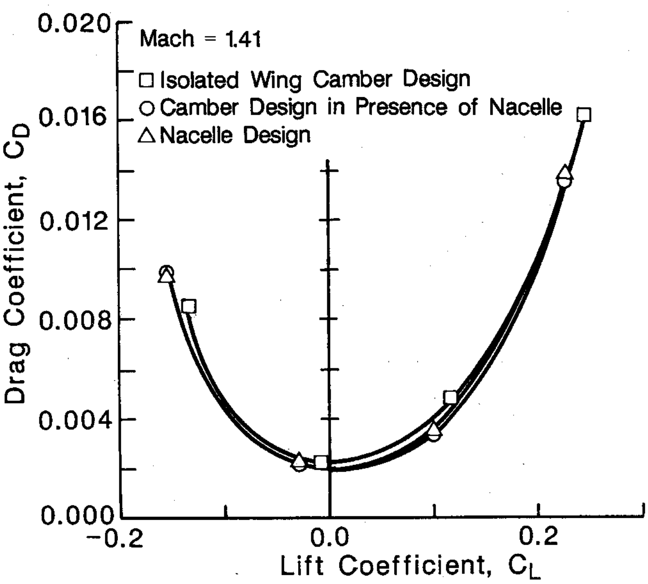

Fig. 13 Drag polars of wing-nacelle configuration.

both designs are shown in Fig. 10. It can be seen that the constrained optimization still achieves a significant drag reduction. However, the reader should be aware that the drag coefficients in this figure are based on the nacelle cross-sectional area. The contribution of this drag reduction to the total airplane drag will be much smaller.

Conclusions and Recommendations

This paper describes a computational method to optimize directly wing camber and nacelle shapes in supersonic flow. All inviscid interference effects between the wing, body, and the nacelle are included in a low-order panel method. However, the internal nacelle flow as well as the exhaust jet/airframe interaction are neglected. Results obtained in the analysis mode agree well with available wind-tunnel data, but some numerical problems are observed for complex configurations. The results of sample design cases agree with findings of other researchers, but the nacelle design is subjected to the numerical problems observed in the analysis mode. It is recommended that the optimization procedure developed in this paper should be implemented into a current panel method code, for example VSAERO for subsonic computations, because the instabilities in the present code are inherent in the analysis code used in the research. Furthermore, supersonic vortex lift at high angles of attack needs to be modeled and included in the analysis code.

References

[1]Suikat, R., "Modification of a Panel Method Computer Program to Optimize the Shape of a Nacelle to Obtain Minimum Configuration Drag in Supersonic Flow," M.S. Thesis, University of Kansas, Lawrence, KS, Oct. 1986; Part 2 – Computer Program Description, University of Kansas Flight Research Lab, KU-FRL-6132-5, to be published. Part 3 – Users Manual, KU-FRL-6132-6, 1987.

[2]Woodward, F.A., "Analysis and Designs of Wing-Body Combinations at Subsonic and Supersonic Speeds," *Journal of Aircraft*, Vol. 5, Nov.-Dec. 1968, p. 528.

[3]Woodward, F.A., "An Improved Method for the Aerodynamic Analysis of Wing-Body-Tail Configurations in Subsonic and Supersonic Flow," Part 1 – Theory and Application, Part 2 – Computer Program Description, NASA CR-2228, May 1973.

[4]Woodward, F.A., Tinoco, E.N., and Larsen, J.W., "Analysis and Design of Supersonic Wing-Body Combinations, Including Flow Properties in the Near Field," NASA CR-73106, Aug 1967.

[5]Roberts, A. and Rundle, K., "Computation of Incompressible Flow about Bodies and Thick Wings Using the Spline Mode System," British Aircraft Corp., Weybridge, UK, May 19, 1972.

[6]Petrie, J.A.H., "A Surface Source and Vorticity Panel Method," *Aeronautical Quarterly*, Vol. 29, Nov. 1978, p. 251.

[7]Shrout, B.L. and Fournier, R.H., "Aerodynamic Characteristics of a Supersonic Cruise Airplane Configuration at Mach Numbers of 2.30, 2.96 and 3.30," NASA TM-78792, Jan 1979.

[8]Maskew, B., "Prediction of Subsonic Aerodynamic Characteristics, a Case for Low Order Panel Methods," *Journal of Aircraft*, Vol. 19, Feb. 1982, p. 157.

[9]Mack, R.J., "A Numerical Method for Evaluation and Utilization of Supersonic Nacelle Wing Interference," NASA TN D-5057, March 1969.